

CT local reconstruction of solid rocket motor based on PI-POCS-TVM

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Abstract: In order to test the defects of solid rocket motor (SRM) accurately and efficiently, the computed tomography (CT) inspection scheme for SRM's defects was investigated and a CT local reconstruction algorithm called prior information-projection onto convex sets-total variation minimization (PI-POCS-TVM) was developed. The SRM was first inspected by industrial CT (ICT) to generate a low-resolution SRM image used as the prior information, then high-resolution local inspection was carried out for SRM's defects. To validate the effectiveness of the CT inspection scheme for SRM's defects, one SRM was inspected by a narrow fan beam ICT system. Filtered back projection (FBP) algorithm, POCS, POCS-TVM and PI-POCS-TVM algorithms were applied to reconstruct CT images. The performance of these algorithms was also compared. Results demonstrated the effectiveness of the CT inspection scheme for SRM's defects. Moreover, the PI-POCS-TVM algorithm has better local reconstruction image quality than the other three algorithms, showing great significance for accurate measurement of SRM's defects.

Key words: interior tomography; exterior tomography; compressed sensing; prior information; projection onto convex sets-total variation minimization (POCS-TVM); solid rocket motor

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Industrial computed tomography (ICT) has played an important role in the nondestructive testing of solid rocket motor (SRM)^[1-3]. How to inspect the SRM's defects, distinguish the type of defects and measure them accurately and fast at the minimum cost? It is a worth studying problem, especially for the SRM with large diameter. For a given ICT system, the reduction of inspecting cost means the reduction of projection data. Fast inspection means real-time image reconstruction. Thus, interior tomography and exterior tomography develop fast in the CT field.

For interior tomography, it aims at recovering the region of interest (ROI) image exactly only with projections associated with lines through the ROI^[4]. However, there is no unique solution to the interior problem. Therefore, researchers have been dedicating to solving the interior problem in the past few decades. Ye, et al.^[5] and Kudo, et al.^[6] demonstrated that there would be an exact and stable solution to the interior problem on the basis of a known sub-region inside an ROI. Later, Li, et al.^[7] solved the interior problem on the condition that

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a sub-region outside an ROI was known, which further relaxed the data sufficiency condition. Yu and Wang^[8] studied the interior problem based on a sparsity model of an ROI, which solved the interior problem exactly and stably without a prior knowledge. In 1997, Rashid-Rarrokhi, et al.^[9] proposed a wavelet-based multiresolution local reconstruction algorithm by using almost completely local data. In the same year, Zhao, et al.^[10] studied wavelet-based local tomography for parallel-beam CT and Katsevich^[11] studied the computation of lambda tomography by generalizing radon transform for limited-angle tomography. In 2006, Yu and Wang^[12] proposed a general formula for exact and efficient fan-beam lambda tomography from data collected along any smooth curve based on even and odd data extensions. In 2009, Quinto, et al.^[13] studied electron lambda-tomography for reconstructing the structure of macromolecular assemblies.

For exterior tomography, Cormack^[14] proposed the exterior reconstruction problem in 1963 and developed an unstable algorithm, then he developed a stable singular value decomposition (SVD) algorithm^[15]. Later, Quinto^[16] improved the SVD algorithm by bounded extrapolation. In 2007, Quinto^[17] extended lambda tomography to exterior and limited angle exterior tomography by compensating exterior data. Nassi, et al.^[18] proposed iterative reconstruction reprojection (IRR) algorithm for exterior reconstruction in 1982. Later Kim, et al.^[19] proposed projection space IRR (PSIRR) algorithm on the basis of IRR. In 2007, Xiong, et al.^[20] studied iterative algorithm for the exterior reconstruction, including IRR algorithm and maximum likelihood-expectation maximization (ML-EM) algorithm. Zeng, et al.^[21] developed the subregion-averaged-TVM-POCS (SA-TVM-POCS) algorithm for the nondestructive testing of pipe in 2010.

On the basis of Yu, et al.^[8] and Zeng, et al.^[21]'s works, the prior information and projection onto convex sets-total variation minimization (POCS-TVM) algorithm are combined to conduct local CT reconstruction of SRM's de-

fects in this paper.

1 CT Inspection scheme of SRM

For the CT inspection of SRM, the usual work is to inspect the key parts of the SRM. The CT inspection scheme^[22] is: first inspect one cross section of the SRM and generate a low-resolution SRM image. If there exist defects in the cross section of the SRM, then high-resolution local inspection is carried out on the local defect area. If it is uncertain whether the cross section of the SRM has defects or not, then the cross section is inspected again and a higher-resolution image is reconstructed. Above procedures proceed until a conclusion about whether the cross section of the SRM has defects or not is drawn.

For the nondestructive inspection of SRM, there are two types of ICT, including the second generation ICT and the third generation ICT. The second generation ICT is narrow fan beam CT, mainly used for the inspection of SRM with small diameter. While the third generation ICT is wide fan beam CT, mainly used for the inspection of SRM with large diameter. The narrow fan-beam ICT is discussed here.

1.1 Full scan of SRM

As shown in Fig. 1, the narrow fan beam ICT adopts translation-rotation (TR) scanning mode. There are N fan beams, and the numbers of fan beams are $1, \dots, N$. X-ray source and detectors translate from one side to the other side of the SRM equispacedly with a large scanning step length, then the SRM rotates a fixed angle

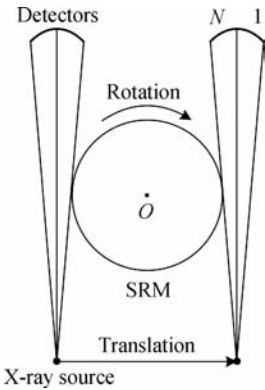


Fig. 1 Sketch of full scan of SRM

around its center of rotation, X-ray source and detectors return to the initial scanning position and scan the SRM again until the projection view reaches 180° . Then a low-resolution SRM image can be reconstructed with the filtered backprojection (FBP)^[23] algorithm.

1.2 Interior local scan of SRM

For the inspection of interior part of SRM, rotation only (RO) scanning mode is adopted. As shown in Fig. 2, adjust the distance between the X-ray source and the center of rotation of SRM to make the local inspection area is within the field of view (FOV). Then SRM rotates around its center of rotation until the projection view reaches 360° .

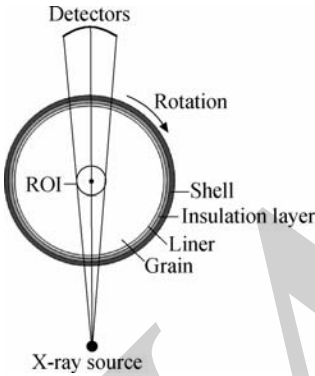


Fig. 2 Sketch of interior local scan of SRM

1.3 Exterior local scan of SRM

The local inspection of exterior part of SRM is the inspection of the bonding interface of SRM. As shown in Fig. 3, adjust the location of

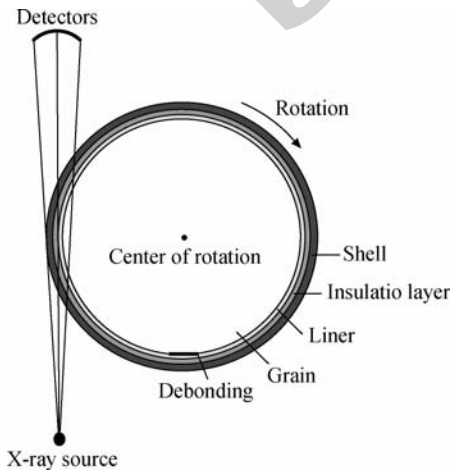


Fig. 3 Sketch of exterior local scan of SRM

X-ray source and detectors to make the bonding interface of SRM is within the FOV. Then SRM rotates around its center of rotation until the projection view reaches 360° .

2 Sparsity analysis

The compressed sensing (CS)^[24] theory is on the basis of sparse representation of signal or image. Though the tomographic image of SRM is non-sparse, but it could be sparsely represented by sparse transforms, for example the discrete gradient transform and wavelet transform. In this paper, the discrete gradient transform is used. As shown in Fig. 4, let the pixel values be labeled by $f_{x,y}$, then the image gradient magnitude^[25] is

$$|\nabla f_{x,y}| = \sqrt{(f_{x,y} - f_{x-1,y})^2 + (f_{x,y} - f_{x,y-1})^2} \quad (1)$$

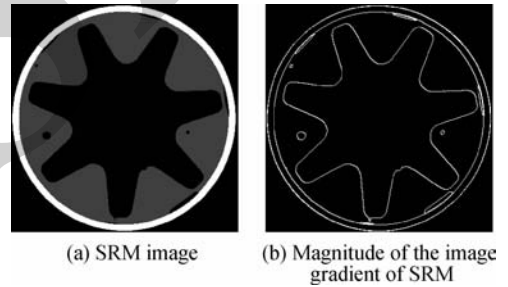


Fig. 4 Discrete gradient transform of SRM

The number of non-zero pixels in the 358×358 SRM image is 53 612, while the number of non-zero pixels in the gradient image is only 4 708. And the total variation (TV)^[25] of the gradient image can be expressed as

$$\|f_{x,y}\|_{tv} = \sum_{x,y} |\nabla f_{x,y}| = \sum_{x,y} \sqrt{(f_{x,y} - f_{x-1,y})^2 + (f_{x,y} - f_{x,y-1})^2} \quad (2)$$

3 PI-POCS-TVM algorithm

On the basis of CS theory, prior information is considered for the local reconstruction of SRM. In this paper, the prior information is the low-resolution SRM image. Let the image be $H \times W$ dimensions, $f_{x,y}$ represent pixel in the context of a 2-D image. The image function is

described by a vector $\mathbf{f}$ of length N_{image} with individual elements $f_i, i=1, 2, \dots, N_{\text{image}}$. where

$$\begin{aligned} i &= (x-1) \cdot W + y \\ x &= 1, 2, \dots, H \quad y = 1, 2, \dots, W \end{aligned} \quad (3)$$

where H and W are the height and width of the 2-D image array, respectively.

The PI-POCS-TVM algorithm consists of two phases; POCS and TVM. The iteration numbers of POCS and TVM phases are labeled by m and n , respectively. The vector $\mathbf{f}^{(\text{pi})}$ represents the low-resolution SRM image, $\mathbf{p}$ the projection data vector with its individual measurement referred to as $p_i, i=1, 2, \dots, N_{\text{data}}$. The system matrix $\mathbf{WM}$ is composed of N_{data} row vectors $\mathbf{WM}_i$ that yield each data point, $p_i = \mathbf{WM}_i \cdot \mathbf{f}$. Different from the POCS-TVM algorithm in Ref. [25], in the “initialization” step of the PI-

$$f_i^{(\text{positivity})}[m] = \begin{cases} f_i^{(\text{pocs})}[m, N_{\text{data}}] & f_i^{(\text{pocs})}[m, N_{\text{data}}] \geq 0 \\ 0 & f_i^{(\text{pocs})}[m, N_{\text{data}}] < 0 \end{cases} \quad (6)$$

2) TVM phase.

(1) Initialization.

$$\mathbf{f}^{(\text{tvm})}[m, 1] = \mathbf{f}^{(\text{positivity})}[m] \quad (7)$$

$$d_a(m) = \|\mathbf{f}^{(\text{pocs})}[m, 1] - \mathbf{f}^{(\text{positivity})}[m]\|_2 \quad (8)$$

(2) TV gradient descent, for $n=2, \dots,$

N_{grad}

$$\mathbf{u}_{x,y}[m, n-1] = \frac{\partial \|\mathbf{f}\|_{\text{tv}}}{\partial f_{x,y}} \bigg|_{f_{x,y} = f_{x,y}^{(\text{tvm})}[m, n-1]} \quad (9)$$

$$\hat{\mathbf{u}}[m, n-1] = \frac{\mathbf{u}[m, n-1]}{\|\mathbf{u}[m, n-1]\|} \quad (10)$$

$$\begin{aligned} \mathbf{f}^{(\text{tvm})}[m, n] &= \mathbf{f}^{(\text{tvm})}[m, n-1] - \\ &\alpha d_a(m) \hat{\mathbf{u}}[m, n-1] \end{aligned} \quad (11)$$

where $n=2, \dots, N_{\text{grad}}$.

(3) Judge the stopping criterion.

Iteration stops if there is

$$\|\mathbf{f}^{(\text{tvm})}[m, N_{\text{grad}}] - \mathbf{f}^{(\text{pocs})}[m, 1]\| \leq \epsilon \quad (12)$$

where ϵ is a small positive number.

Otherwise, initialize the next loop.

$$\mathbf{f}^{(\text{pocs})}[m+1, 1] = \mathbf{f}^{(\text{tvm})}[m, N_{\text{grad}}] \quad (13)$$

4 Experiments

To validate the effectiveness of the CT inspection scheme of SRM's defects and the PI-POCS-TVM algorithm, experiments were carried out. A second generation narrow fan beam ICT was used to test one SRM. First, one cross

POCS-TVM algorithm, the initial condition is $\mathbf{f}^{(\text{pocs})}[m, 1] = \mathbf{f}^{(\text{pi})}$ instead of $\mathbf{f}^{(\text{pocs})}[m, 1] = \mathbf{0}$.

On the basis of POCS-TVM algorithm^[25], the main steps of the PI-POCS-TVM algorithm are as follows:

1) POCS phase.

(1) Initialization.

$$m = 1 \quad \mathbf{f}^{(\text{pocs})}[m, 1] = \mathbf{f}^{(\text{pi})} \quad (4)$$

(2) Projection data update by ART algorithm.

$$\begin{aligned} \mathbf{f}^{(\text{pocs})}[m, n] &= \mathbf{f}^{(\text{pocs})}[m, n-1] + \\ &\lambda \frac{p_i - \mathbf{WM}_i \cdot \mathbf{f}^{(\text{pocs})}[m, n-1]}{\mathbf{WM}_i' \cdot \mathbf{WM}_i} \mathbf{WM}_i \end{aligned} \quad (5)$$

where $n=2, \dots, N_{\text{data}}$; λ denotes the weighted factor.

(3) Non-negativity constraint.

section of the SRM was inspected with the scanning step length $d=2$ mm and a low-resolution image was reconstructed, defects of the SRM can be detected in the low-resolution SRM image. Then, one interior defect area and the bonding interface of the SRM for further high-resolution local inspection were chosen. At last, the cross section of the SRM was inspected again with the scanning step length $d=0.5$ mm and a high resolution image was reconstructed to compare it with the reconstructed local CT image. For the given ICT system, the number of detector bins is 32, the total fan angle is 10° , and the number of projection views is 360.

4.1 Interior reconstruction

FBP, POCS, POCS-TVM and PI-POCS-TVM algorithms were applied to reconstruct the interior part of SRM. The iteration numbers of POCS and TVM were 100 and 10, respectively. Reconstructed images are shown in Fig. 5. Horizontal and vertical profiles along the center of SRM images are presented in Fig. 6.

From Fig. 5 and Fig. 6, it is clearly that the image quality reconstructed by our proposed PI-POCS-TVM algorithm is better than those re-

constructed by FBP, POCS and POCS-TVM algorithms. There are truncated artifacts in the image reconstructed by FBP algorithm. For the

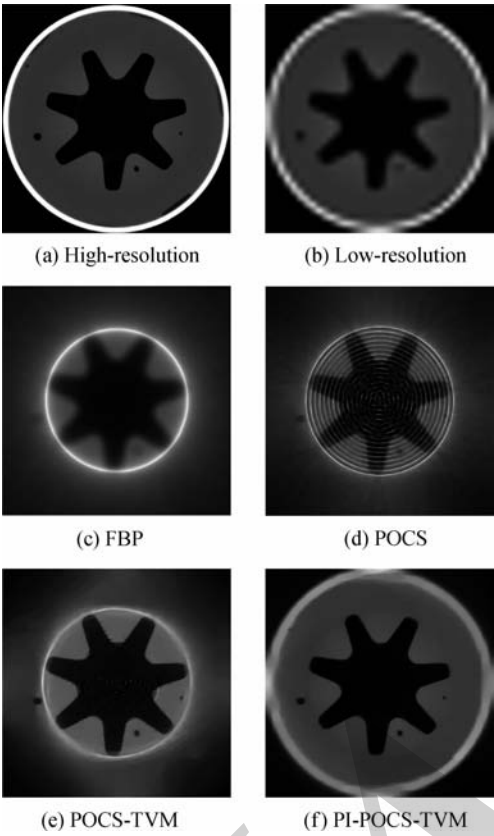


Fig. 5 Interior reconstructed results of SRM

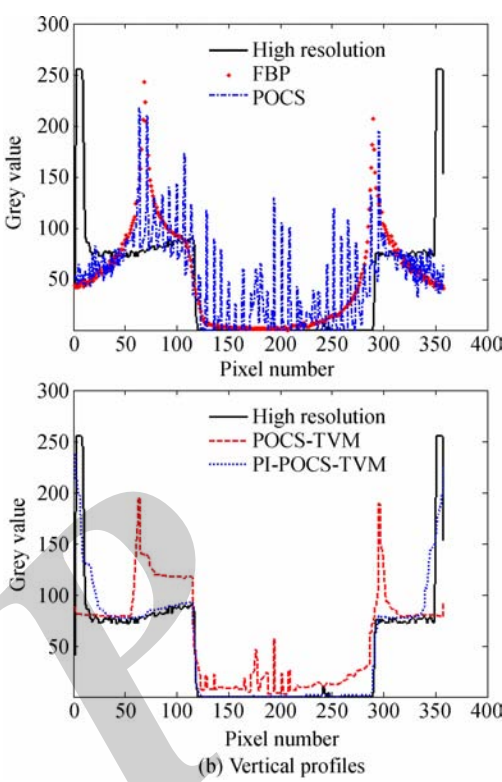
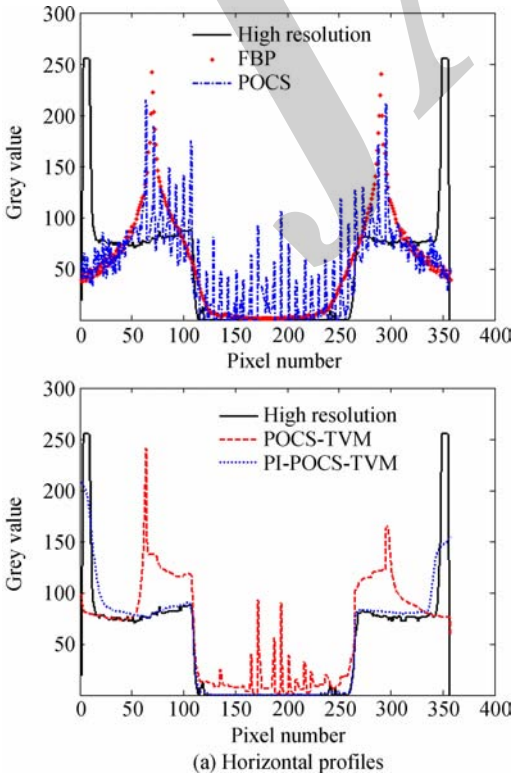


Fig. 6 Relative profiles along the center of SRM images in Fig. 5

image reconstructed by POCS algorithm, there are ring artifacts in the ROI. And there are truncated artifacts in the image reconstructed by POCS-TVM algorithm too. While in the image reconstructed by PI-POCS-TVM algorithm, there is no artifact in the ROI of SRM.

4.2 Exterior reconstruction

Also, FBP, POCS, POCS-TVM and PI-POCS-TVM algorithms were used to reconstruct the exterior part of SRM. The iteration numbers of POCS and TVM were 100 and 10, respectively. Reconstructed results are shown in Fig. 7. Horizontal and vertical profiles along the center of SRM images are shown in Fig. 8.

From Fig. 7 and Fig. 8, it can be seen that the bonding interface of the SRM is hard to see in the image reconstructed by FBP. There are shading artifacts in the images reconstructed by POCS and POCS-TVM algorithms. While in the image reconstructed by PI-POCS-TVM algorithm, the debonding defects can be seen clearly.

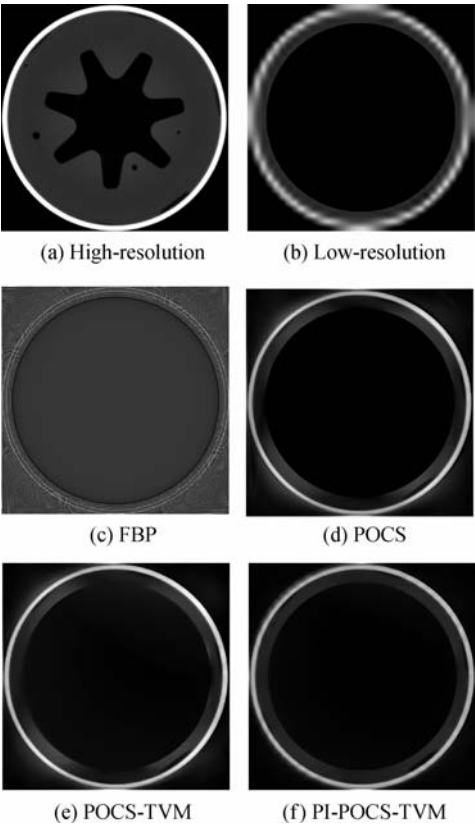


Fig. 7 Exterior reconstructed results of SRM

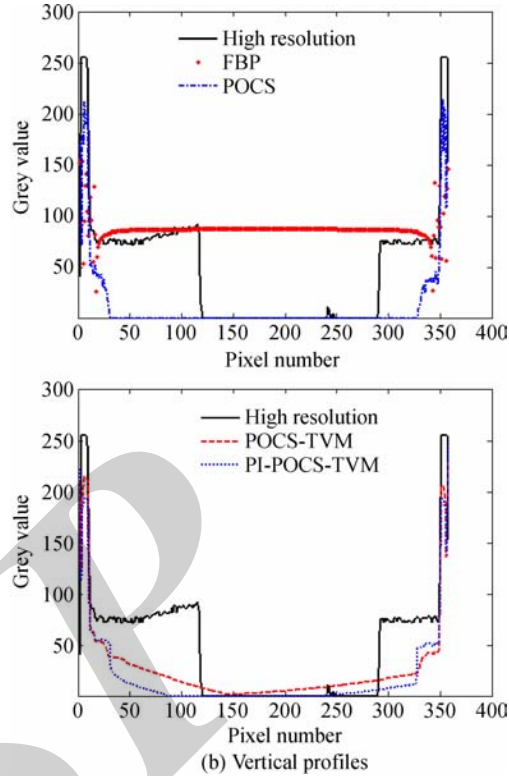


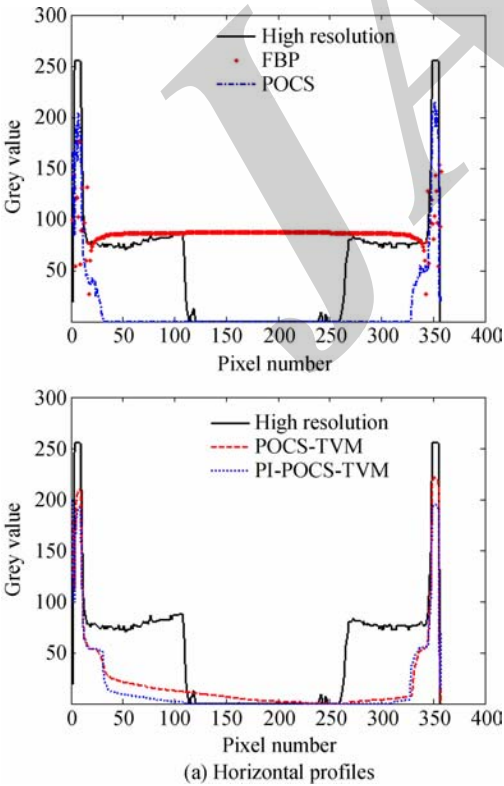
Fig. 8 Relative profiles along the center of SRM images in Fig. 7

5 Conclusions

- 1) The CT image of SRM is non-sparse, but the discrete gradient image of SRM is sparse, therefore CT reconstruction algorithm based on compressed sensing theory can be applied to the reconstruction of SRM image.
- 2) Compared with POCS-TVM algorithm, the proposed PI-POCS-TVM algorithm uses the reconstructed low-resolution image instead of vector zero as the initial condition of POCS-TVM algorithm, which greatly improves the reconstructed CT image quality of SRM’s defects. This lays a solid foundation for the accurate measurement of SRM’s defects.

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(a) Horizontal profiles

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